$6d \to 5d$

Take $\Phi_i^I \to \frac{1}{2\pi R} \varphi_i^I$, $\quad H_i \to \frac{1}{2\pi R}\left(f_i \wedge dx^5 + *^{(5)} f_i\right)$

where $x^5 \sim x^5 + 2\pi R$ parametrizes the circle S^1_R and $*^{(5)}$ denotes the 5d Hodge-operation

$\to H_i = *H_i$

$\to$ obtain 6d action from 5d by uplift:

$$-\frac{\pi R}{g^2} \Omega_{ij}\left(H_i \wedge *H_j + \sum_{I=1}^{5} \partial_\mu \Phi_i^I \partial^\mu \Phi_i^I\right) + (\text{Fermions})$$

$$\subset \mathcal{L}_{\text{tensor}}$$

Kinetic terms determine 6d Dirac pairing

$$dH_i = q_i \delta_{\Sigma_2} \iff q_i = \int_{\Sigma_3} H_i ,$$

where Σ_2 is linked by Σ_3

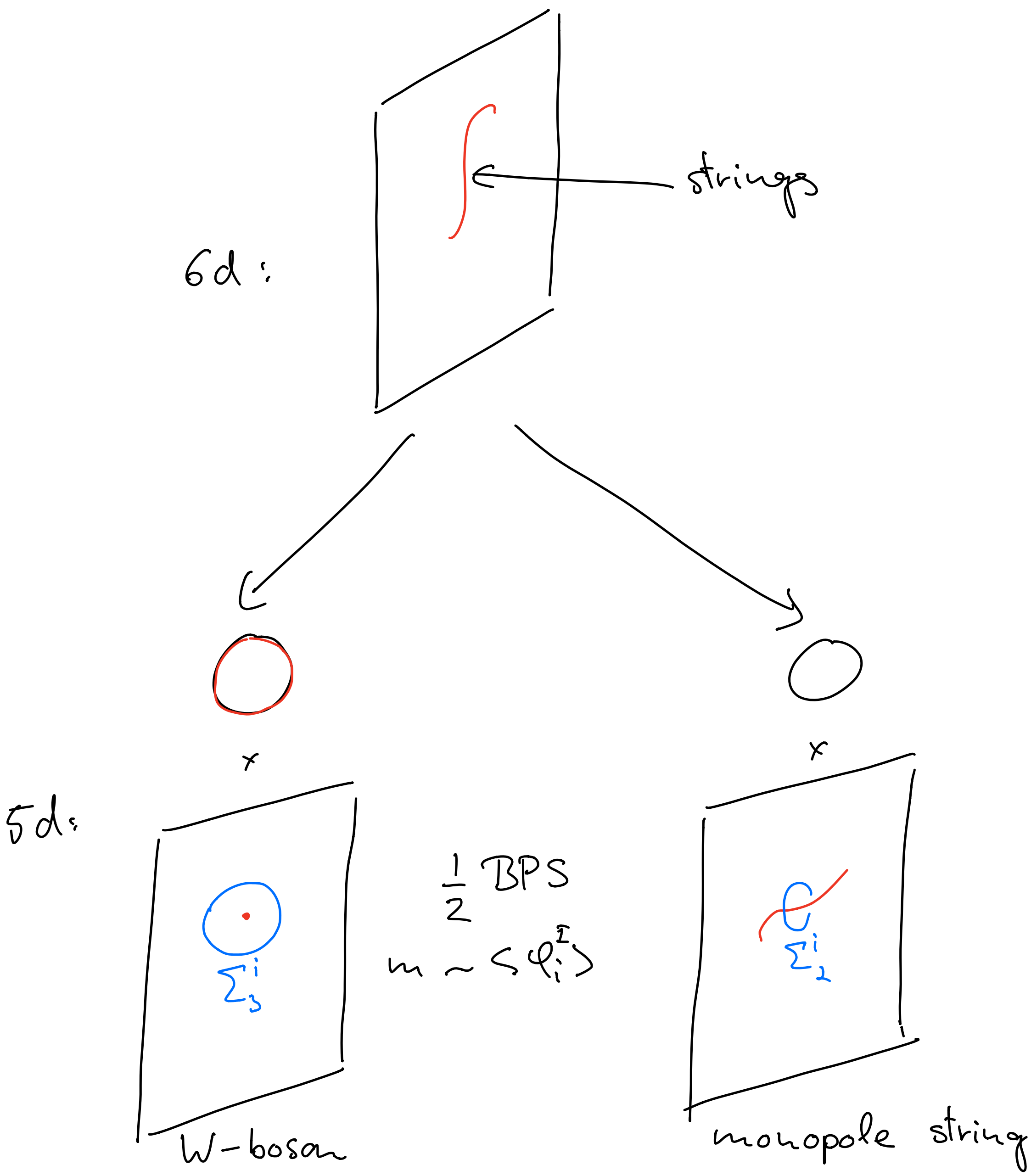
$\to$ integer-valued Dirac-pairing between two strings:

$$\frac{R}{g^2} \Omega_{ij} q_i q'_j \in \mathbb{Z}$$

$\xrightarrow{S^1_R}$ 5d: correspond to BPS states on Coulomb branch

electrically charged W-bosons
correspond to roots $\alpha \in \Delta_{\mathfrak{g}}$

magnetically charged monopole strings
correspond to coroots h_α



$\rightarrow$ constraints on 5d theory:

Consider W-boson corresponding to α_i

Then $[h_j, e_{\pm i}] = \pm C_{ij} e_{\pm i}$

$\rightarrow$ have charge C_{ij} with respect to A_j:

(a) $$(e_i)_j = C_{ij} = \frac{\Omega_{jk}}{g^2} \int_{\Sigma_3^i} * f_k$$

Similarly, magnetic charges are given by

(b) $$(m_i)_j = \delta_{ij} = \frac{1}{2\pi} \int_{\Sigma_2^i} f_j$$

Obtain (a) and (b) by integrating 3-form flux H_i over Σ_3^i and $\Sigma_2^i \times S^1_R$ respectively

$$\rightarrow (q_i)_j = 2\pi \delta_{ij}, \quad C_{ij} = \frac{4\pi^2 R}{g^2} \Omega_{ij}$$

Since $\Omega_{ij} = \mathrm{Tr}_{\mathfrak{g}}(h_i h_j) \rightarrow \Omega_{ij}$ is symmetric

same must be true for Cartan matrix

C_{ij} ! $\rightarrow$ $\mathfrak{g}$ is "simply-laced"

Non-renormalization theorems:

Discuss constrains on $\mathcal{L}_{tensor}$ following from supersymmetry.

Recap:

$$\mathcal{L}_{free} = -\frac{1}{2}\sum_{I=1}^{5}(\partial_\mu \phi^I)^2 - \frac{1}{2} H \wedge * H + (\text{Fermions})$$

Introduce fields:

$$\Psi = \left(\sum_{I=1}^{5} \phi^I \phi^I\right)^{\frac{1}{2}}, \qquad \hat{\Phi}^I = \frac{\Phi^I}{\Psi},$$

"radial" "transverse"

Ψ has dimension 2, $\hat{\Phi}^I$ are dimension-less

→ activate vev $\langle \Psi \rangle$

→ break conformal symmetry and $SO(5)$ R-sym to $SO(4)_R$

Ψ is the Goldstone boson of conformal sym. breaking

$\hat{\Phi}^I$ are Goldstone bosons of R-symmetry breaking

Note: $\sum_{I=1}^{5} \hat{\Phi}^I \hat{\Phi}^I = 1$

→ $\hat{\Phi}^I$ describe a unit $S^4 = SO(5)_R / SO(4)_R$

Activating $\langle \hat{\Phi}^I \rangle$, some fields acquire mass $\sim \sqrt{\langle \Psi \rangle}$

$$\underset{}{\mathcal{T}_{\mathfrak{g}}} \longrightarrow \underset{\text{SCFT}}{\mathcal{T}_{\mathfrak{h}}} + \underbrace{\text{ATM}}_{\text{contains bosons } \varphi \text{ and } \hat{\Phi}^I}$$

Integrating out massive fields gives:

$$\mathcal{L}_{\text{tensor}} = \mathcal{L}_{\text{free}} + \underbrace{\sum_i f_i(\Phi^I)\, \mathcal{O}_i}_{\substack{\text{constrained by} \\ \text{conformal and R-sym.} \\ +\ (2,0)\ \text{SUSY}}}$$

Expand all coefficient functions $f_i(\Phi^I)$ in fluctuations around a fixed vev

$$\Phi^I = \langle \Phi^I \rangle + \delta\Phi^I ,$$

$$f_i(\Phi^I) = f_i\big|_{\langle\Phi\rangle} + \partial_I f_i\big|_{\langle\Phi\rangle} \delta\Phi^I + \frac{1}{2}\partial_I \partial_J f_i\big|_{\langle\Phi\rangle} \delta\Phi^I \delta\Phi^J$$

$$+ \cdots$$

constitute irrelevant deformations of $\mathcal{L}_{\text{free}}$

Use classification of (2,0) SCFT deformations studied in § 2.2 $\longrightarrow$ F- and D-terms:

- F-terms:

$$\mathcal{L}_F = Q^8\Big(\underbrace{\Phi^{(I_1} \cdots \Phi^{I_n)} - (\text{traces})}_{\substack{\frac{1}{2}\text{BPS} \\ \text{traceless, sym. } (n-4)\text{-tensor of } SO(5)_R}}\Big) , \quad (n \geq 4)$$

contains 4 derivatives

- D-terms:

$$\mathcal{L}_D = Q^{16}\mathcal{O},$$

where $\mathcal{O}$ is Lorentz scalar.

contains 8 derivatives

Consider for example

$$f_2(\Phi^I)(\partial\Phi)^2 \longrightarrow \left(f\big|_{\langle\Phi\rangle} + \partial_I f_2\big|_{\langle\Phi\rangle}\,\delta\Phi^I + \dots\right)(\partial\Phi)^2$$

$\partial_I f_2\big|_{\langle\Phi\rangle}$ multiplies 2-derivative interaction of 3 scalars

→ ruled out as # derivatives < 4

→ $\partial_I f_2(\Phi_I)\big|_{\langle\Phi\rangle} = 0$ → f_2 is constant

Consider

$$f_4(\Phi^I)(\partial\psi)^4 \to \left(f_4\big|_{\langle\Phi\rangle} + \partial_I f_4\big|_{\langle\Phi\rangle}\delta\Phi^I + \tfrac{1}{2}\partial_I\partial_J f_4\big|\,\delta\Phi^I\delta\Phi^J + \dots\right)(\partial\psi)^4$$

$f_4\big|_{\langle\Phi\rangle}(\partial\psi)^4$ and $\partial_I f_4\big|_{\langle\Phi\rangle}(\partial\psi)^4$ are F-terms

R-sym. inv. R-sym. vector

trace $\delta^{IJ}\partial_I\partial_J f_4\big|_{\langle\Phi\rangle}(\partial\psi)^4$ is 4-derivative term with 6 fields <u>but</u> R-sym. inv.

$\rightarrow$ ruled out ! $\rightarrow$ $\delta^{IJ} \partial_I \partial_J f_4(\Phi^K) = 0$

$$\rightarrow f_4(\Phi^I) = \frac{b}{\varphi^3}$$

b in fact controls <u>all</u> four-derivative terms in $\mathcal{L}_{\text{tensor}}$!